Harpsichord & fortepiano

Vol. 5, No. 3 October, 1995

© Peacock Press.
Licensed under CC BY-NC 4.0.
You are free to share and adapt the content for non-commercial purposes, provided you give appropriate credit to Peacock Press and indicate if changes were made. Commercial use, redistribution for profit, or uses beyond this license require prior written permission from Peacock Press.

Musical Instrument Research Catalog
(MIRCat)

The Cent System

WITH AN EASY METHOD OF CALCULATION

Carl Sloane

THE CENT, which is a commonly used unit of interval size, is defined as 1/1200 of an octave or 1/100 of a semitone in equal temperament. Invention of the cent system is generally credited to Alexander Ellis, today perhaps best remembered as the translator of Helmholtz's *Lehre von den Tonempfindungen*, although he appears to have been anticipated by several others in some or all respects. A list of such systems is given in fn. 1—arguably the best modern book on temperament. Cents are used because they are easier to manipulate than frequency ratios—addition replaces multiplication and division replaces extraction of roots—and because of their greater descriptive power.

Many harpsichordists will be familiar with tables for use with electronic tuners, such as that given opposite, which show the deviation in cents from the pitch in equal temperament. If the tempering of the fifths in cents is known, there is little difficulty in constructing one of these tables, the only additional requirement being the cent value of the true fifth (702·0). To take an example from Werckmeister III, the fifths between C and A are each narrowed by $^1/_4$ Pythagorean comma or about six cents, so that C will be: – (702 – 6) x 3 + 2400 = 312 cents above A, or 12 cents above its equal-tempered pitch—a minus sign precedes the parentheses because the fifths are tuned downward from A. A table using A as the starting point can be converted to one on C, for example, by in effect transposing the whole line to the left by 312 cents. Every note above C will thus have 312 cents deducted from its cent reading, and each note below will have (1200 – 312) or 888 cents added to it. The deviations will in general all be different in the transposed version.

In a case such as Werckmeister III, the cent values of the fifth and Pythagorean comma can be merely looked up in a table of standard intervals, such as that of Ellis, but tables, even when they are available, are cumbersome to use, and often inadequate.[2] This inadequacy can be illustrated with Werckmeister's septenarius temperament (no. VI), which is given in terms of string lengths.[3] To examine the properties of the temperament, as distinct from the rationale behind it, the most logical method is to convert the string ratios to cents, but as most of the ratios represent tempered intervals, they will not be found in tables and must be converted manually. A table of cent values of the various intervals will convey much more to the imagination than Werckmeister's table, and a meaningful graphical depiction would scarcely be possible with Werckmeister's numbers.[4]

Cents can also be used in the calculation of *beat rates*. If the tempering in cents of the fifth, major third and major sixth is known, the beat rate can be estimated easily and accurately by simply multiplying the tempering by the interval ratio of the fundamental above or below a reference tone (at Baroque pitch, the reference notes are $E_5\flat$ for fifths, and $F_4\sharp$ for thirds and sixths).[5] The formal method of beat-rate determination requires calculation of note frequencies, which in turn generally requires extraction of roots, or at least a considerable amount of arithmetic if an approximative method is used.

Cent calculation

JUST as the frequency ratio of an equal-tempered semitone is $2^{1/12}$, the frequency ratio of an interval of one cent is $2^{1/1200}$ and that of an interval of x cents is $2^{x/1200}$. To calculate the size in cents of an interval of frequency ratio r, it is necessary to solve the equation

$$2^{x/1200} = r$$

On taking logarithms of both sides and rearranging, this equation becomes

$$x = 1200 \log r / \log 2$$

For the casual user the calculation of logarithms is liable to pose a problem. The method presented below, which is really just an algebraic method of approximating logarithms, skirts this difficulty, although a basic calculator will be required.

The frequency ratio r will be in the form either of a rational fraction, say m/n, or of a decimal fraction (n equal to 1). The first step is to calculate the quantity

$$p = (m + n)/(m - n)$$

—if the fraction is less than 1, the denominator will have a negative sign, which should be ignored. When p has been determined, it should be stored in the calculator's memory. It is then only necessary to substitute the value for p into the formula

$$3462{\cdot}5/(p - 1/3p) \qquad \boxed{1}$$

to obtain the desired cent value. There are, however, a few points in regard to this last operation worth mentioning. Since the calculator's memory is already occupied, it is convenient to calculate the denominator first, commencing with the term $1/3p$. Subtraction of p from this quantity gives a number with a negative sign, which should again be ignored. At this point the calculator's reciprocal function (often the key sequence "÷ =") should be used. If the calculator has no reciprocal function, the value of $p - 1/3p$ can be rounded to five figures, written down and then keyed back in once the numerator has been entered, but there is little point in writing down more than five figures, as the factor 3462·5 has already been rounded, and there is an inherent deficit in the formula in any case. The method is correct to four places for intervals up to and including the augmented fourth or tritone—45/32 = 590·2 cents—but a larger interval should be treated as the inversion of a smaller interval whose cent value can be subtracted from 1200 or, as a last resort, as a combination of two smaller intervals. For most purposes, such as temperament reconstruction etc., four-place accuracy is adequate. If only three figures are required, the numerator can be rounded to 3462, not 3463. Again, the interval should not exceed the tritone.

A second method, which works well only with intervals of a tone or less, uses the following formula:

$$3462{\cdot}5/p \qquad \boxed{2}$$

For a tone (9/8) there will be an error of 2 in the fourth place, but for a semitone (16/15) the answer rounds correctly to four places.

For the basis of formulas 1 and 2, see fn. 6.

Table: Cent deviations for the Werckmeister III temperament, for use with electronic tuners

A	B♭	B	C	C♯	D
0	8	4	12	2	4
E♭	**E**	**F**	**F♯**	**G**	**G♯**
6	2	10	0	8	4

Bibliography

[1] *Acustica Accordatura e Temperamento nell' Illuminismo Veneto,* Patrizio Barbieri (Rome, 1987), pp. 278–89

[2] *On the Sensations of Tone,* Hermann Helmholtz, trans. Alexander Ellis, with additions by translator (n.p., 1885), 1954 reissue, pp. 453–56

[3] *Musicalische Temperatur,* Andreas Werckmeister (Quedlinburg, 1691), 1983 reissue, pp. 71–74

[4] *The Mysterious Werckmeister VI Temperament,* Carl Sloane, *The Diapason* (Oct. 1990), p. 15

[5] *A Simplified Method of Beat Rate Calculation,* Carl Sloane, *The Diapason* (June 1987), p. 2

[6] *Die Lehre von den Kettenbrüchen,* Oskar Perron (Stuttgart, 1957), 1977 reissue, p. 155

SASSMANN

Meisterwerkstätten
für historische Tasteninstrumente

Cembalobau · Hammerflügel · Virginale · Spinette · Clavichorde

Instrumente erlesener Qualität, Tradition und Innovation

42499 Hückeswagen - Wiehagen · Tel: 02192/4031